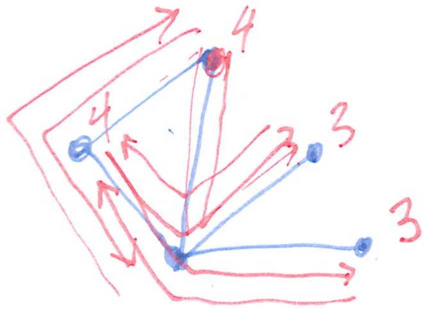
Clustering Coefficient.

$$CC = \frac{6 \cdot \#\text{triangles}}{\#\text{ paths of length } 2}.$$

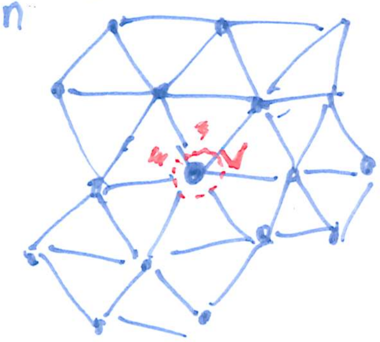


#trngles = 1.

#paths of length 2 = 14

Triangular Lattice

n vertices



approximation – "ignore boundary"

"assume that behaviour same if we look at internal vertex".

#triangles.

$= \frac{6n}{3} = 2n$

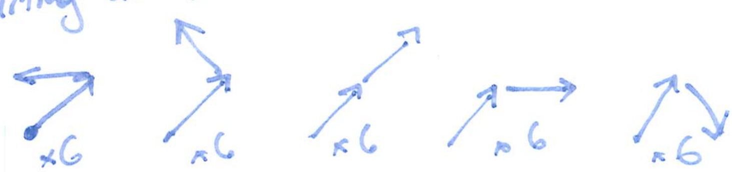
#triangles

6 triangles incident with each vertex.

Overcount by factor of 3

Because each Δ counted 3 times.

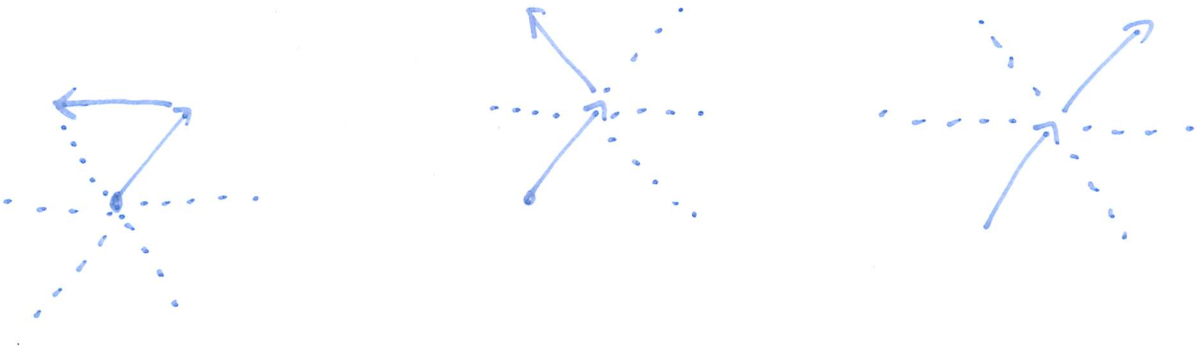
#paths of length two starting at v

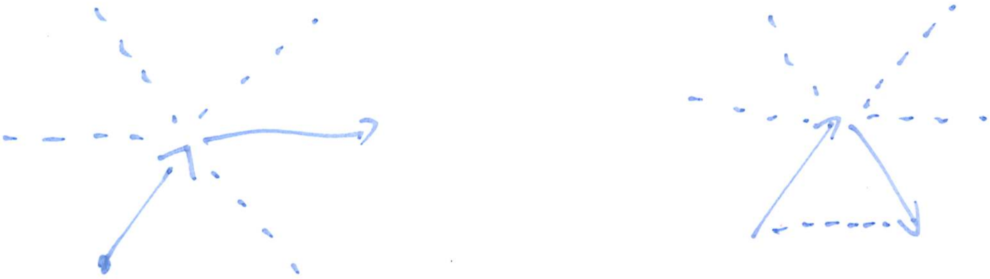


= 30

#paths of length two = 30n

$$CC = \frac{12n}{30n} = 0.4.$$





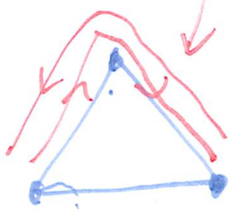
$\frac{2}{5}$ cases

0.4

also an edge.

$$CC = \frac{\#\text{ two paths (which are closed)}}{\#\text{ two paths}}$$

$$= \frac{6 \cdot \cancel{\#\text{triangles}}}{\#\text{ two paths}}.$$

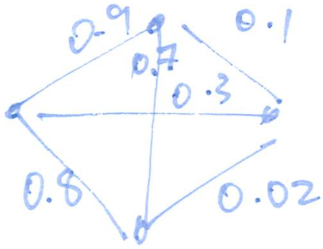


6 closed paths of length 2

Random Graph Process

n vertices

on each edge place
- rand()
- uniformly chosen real in [0, 1].
- "time" at which edge appears.



$t=0$ $t=0.02$ $t=0.1$ . . . . . .

stop at time t:

- each edge has probability t of being present.

$\in [0,1]$

Erdös - Renyi

- n vertices
- each pair joined indep. with probability p.

<u>Degree</u>.

expected degree

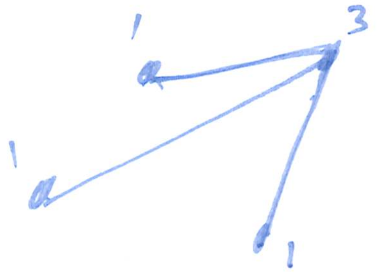
$c = (n-1)p$

<u>Clustering coefficient</u> $= \frac{\# \text{triangles}}{\# \text{two paths}} = \frac{n^3 p^3}{n^3 p^2} \sim p.$

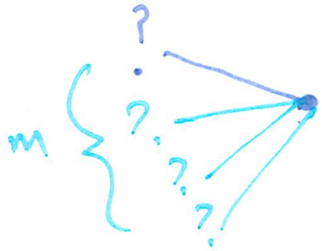
Preferential attachment m edge model

time step i :

add vertex and edge



present graph



prob joins vertex i

$$= \frac{d_i}{\sum_i d_i}$$

Pro: • "realistic", power-law degree distribution

• analysis not too bad.