Modelling Complex Systems

Self-propelled particles

This lecture is adapted from Vicsek, T. & Zafeiris, A. (2012) Collective Motion..

See: arXiv:1010.5017v2




Why do animals move together?

- Increased accuracy (many estimates)
- Increased awareness (many eyes)
- Confuse predators and reduce encounters

How do animals move together?

- Group formation usually seems to be *spontaneous.*
- Based on local interactions
- Phenomenological models
- Can ignore 'first principles' physics!
  e.g. Conservation of momentum
- Use biological principles and limits instead.

Random walk in one dimension

- Run 'RandomWalk1D'

$$x_i(t+1) = x_i(t) + v_0 \, u_i(t)$$

$$u_i(t+1) = a u_i(t) + e_i(t)$$

future position: $x_i(t+1)$; current position: $x_i(t)$; current velocity: $u_i(t)$

future velocity: $u_i(t+1)$; current velocity: $u_i(t)$; stochastic effect: $e_i(t)$

$e_i(t)$ *is a* random number selected uniformly at random from a range $[-\eta/2, \eta/2]$

Attraction in one dimension

- Run 'Aggregate1D'

future position — current position — current velocity

$$x_i(t+1) = x_i(t) + v_0\, u_i\,(t)$$

$$u_i(t+1) = a u_i(t) + (1-a)\, s_i(t) + e_i(t)$$

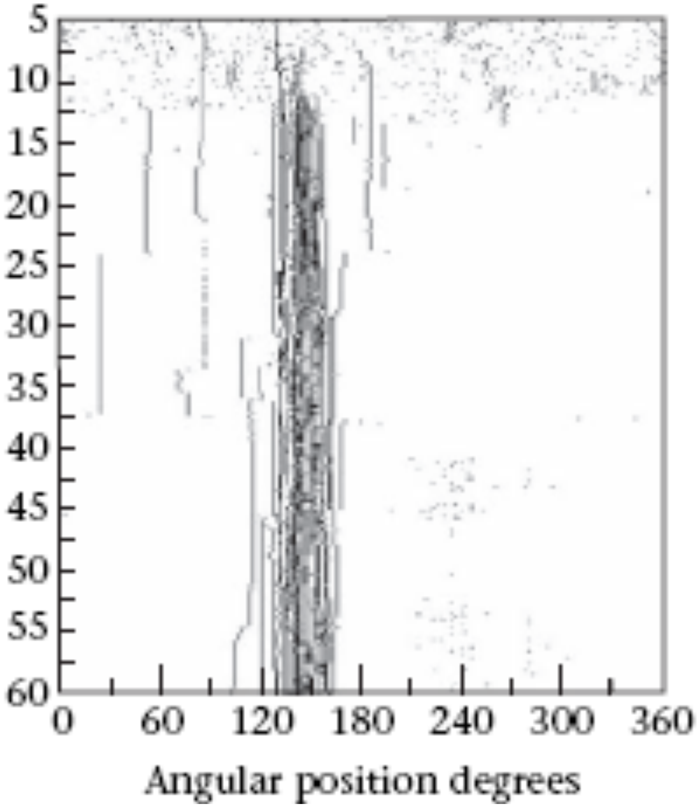
future velocity — current velocity — Direction to most neighbours — stochastic effect

$$s_i(t) = \frac{1}{|R_i|} \sum_{j \in R_i} \text{sign}\Big(x_i(t) - x_j(t) \Big)$$

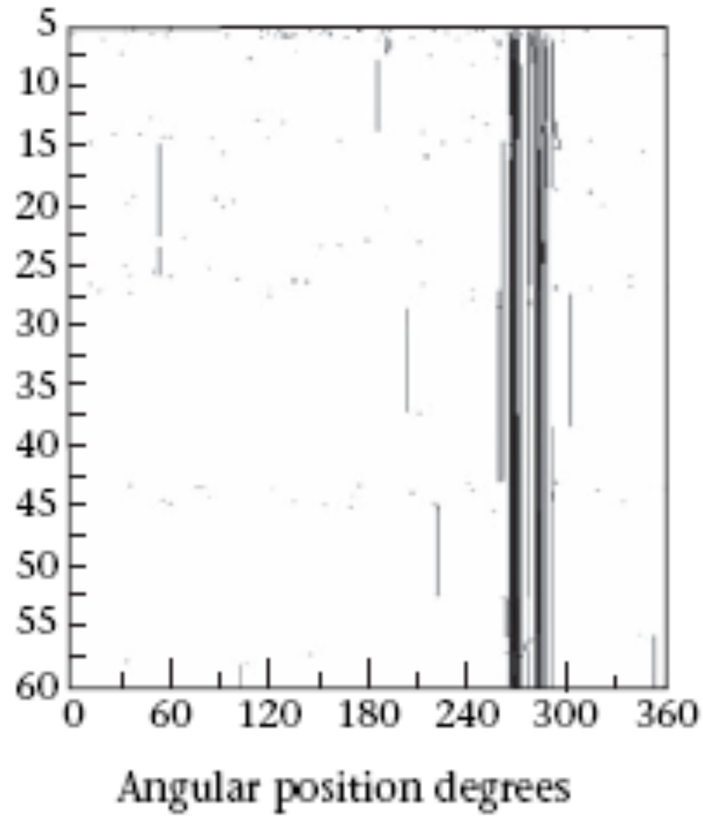
$e_i(t)$ *is a* random number selected uniformly at random from a range $\left[-\eta/2, \eta/2\right]$

Cockroach aggregation

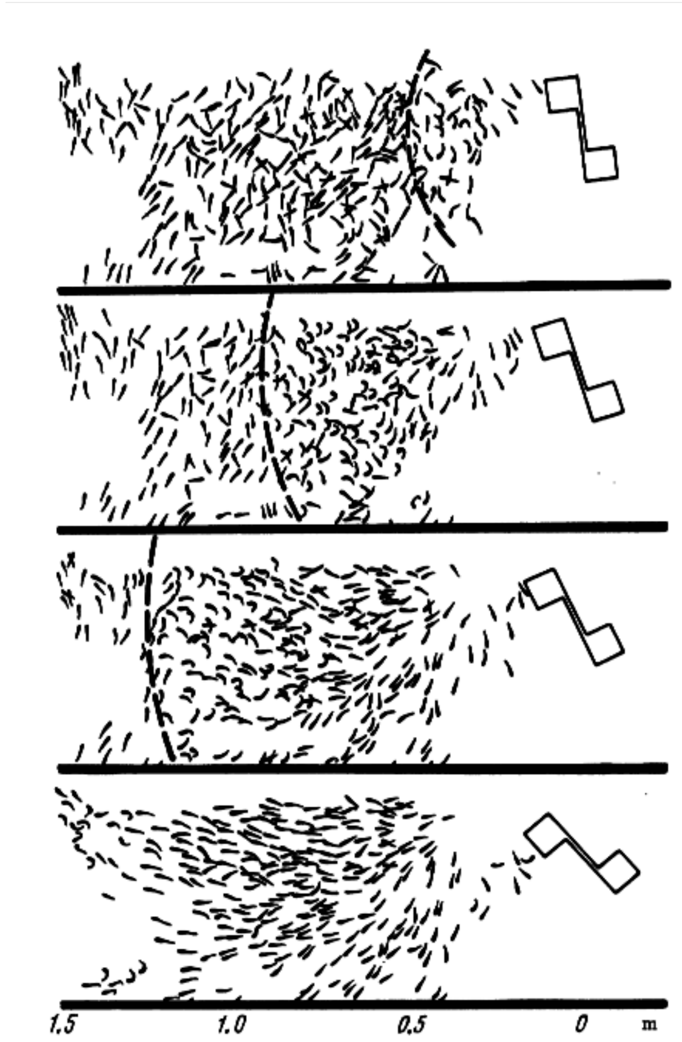
Cockroaches



Model



Radakov's fish



Alignment model in one dimension

- Run 'Align1D'

future position — current position — current velocity

$$x_i(t+1) = x_i(t) + v_0\, u_i\,(t)$$

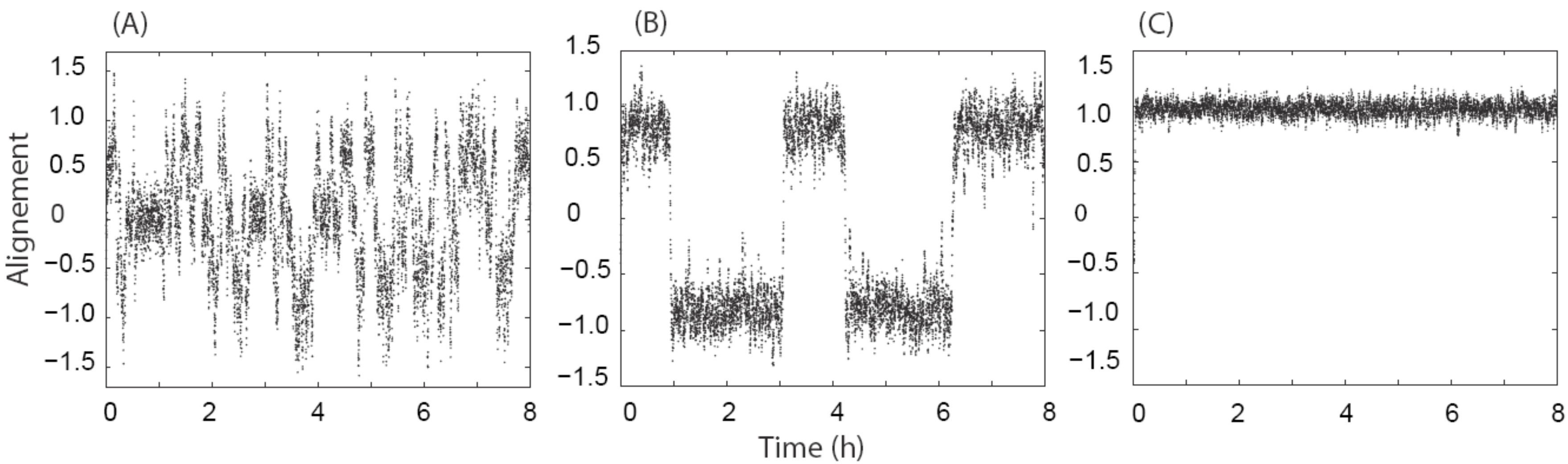
$$u_i(t+1) = au_i(t) + (1-a)\, s_i(t) + e_i(t)$$

future velocity — current velocity — velocity of neighbours — stochastic effect

$$s_i = G\left(\frac{1}{|R_i|}\sum_{j\in R_i} u_j(t)\right) \qquad G(u) = \begin{cases}(u+1)/2 & for \quad u>0\\ (u-1)/2 & for \quad u<0\end{cases}$$

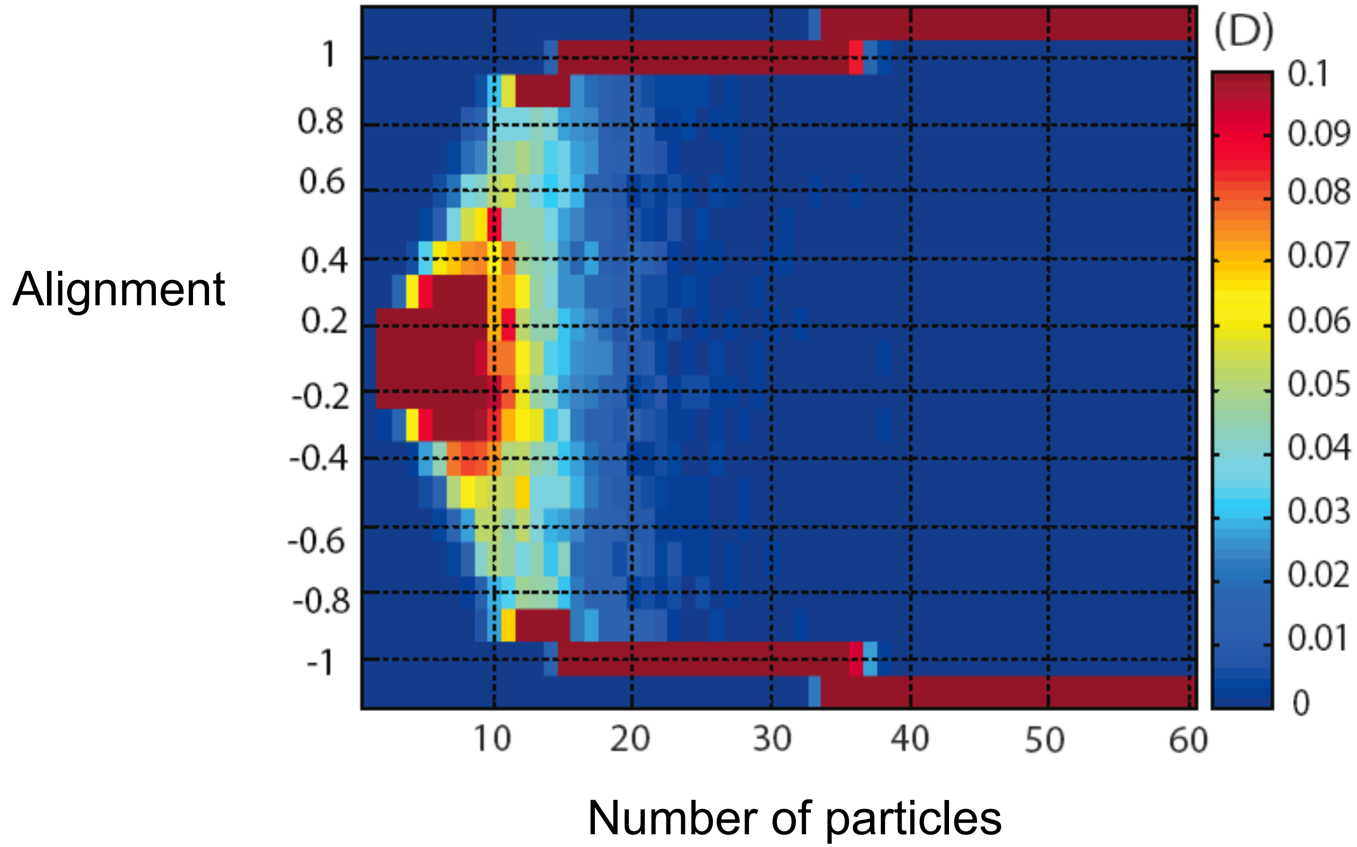
e is a random number selected uniformly at random from a range $[-\eta/2, \eta/2]$

Alignment



$$\phi = \frac{1}{n}\sum_{i=1}^{n} \underline{u}_i(t)$$ measures order in the system.

1D self-propelled particles



$$\phi = \frac{1}{n}\sum_{i=1}^{n} \underline{u}_i(t)$$ measures order in the system (alignment).

Buhl et al. (2006), *Science*
Yates et al. (2009), *PNAS*

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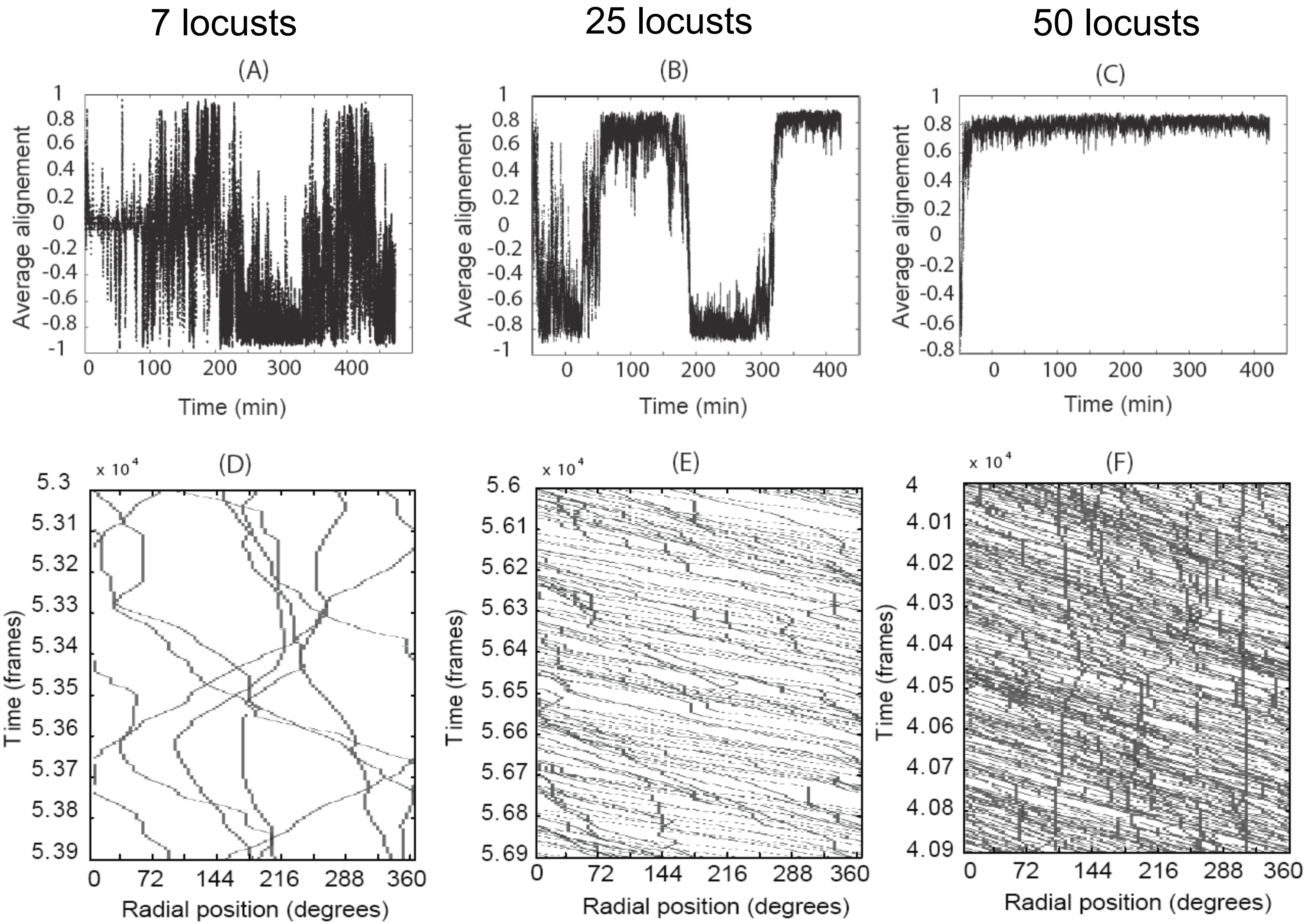
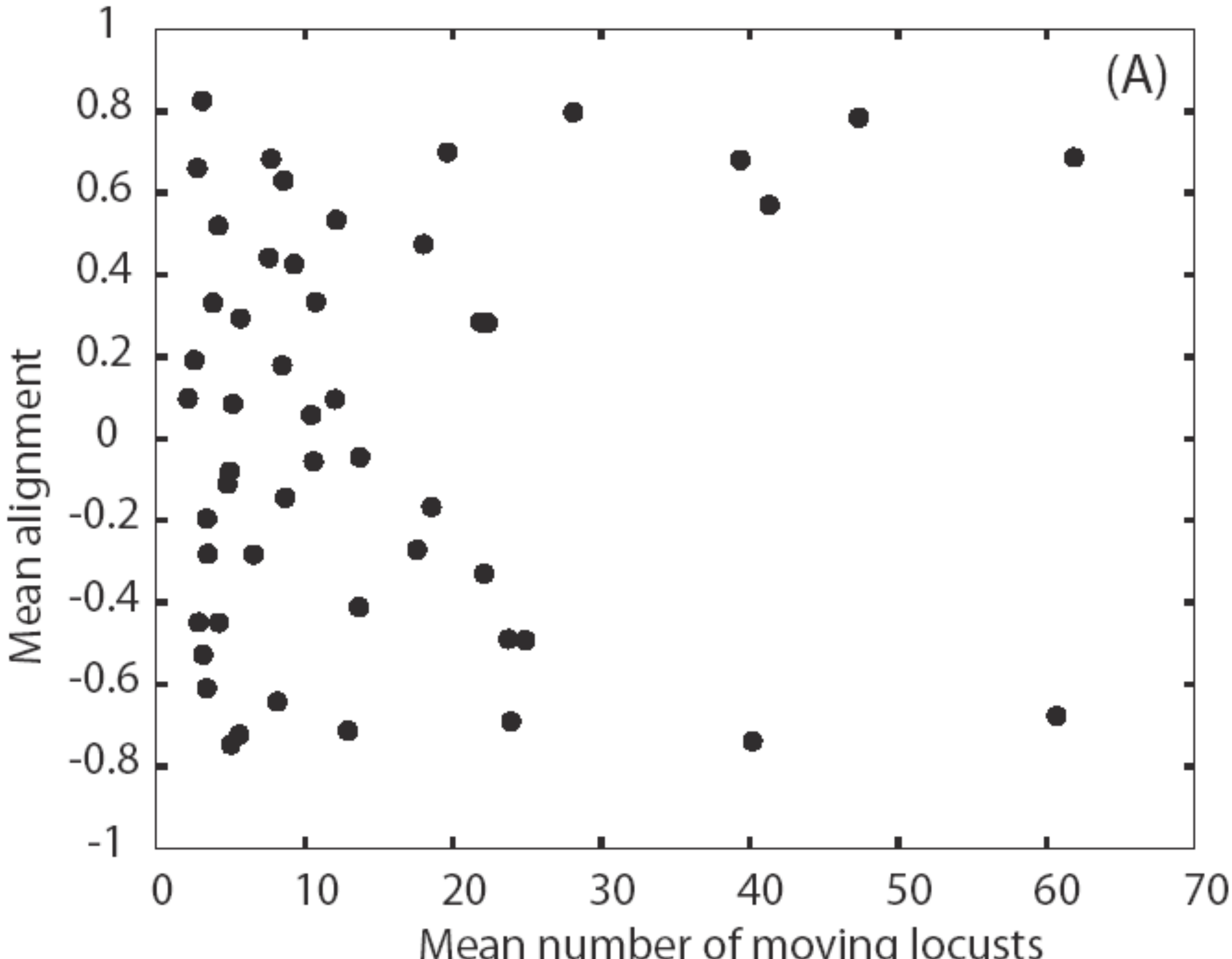
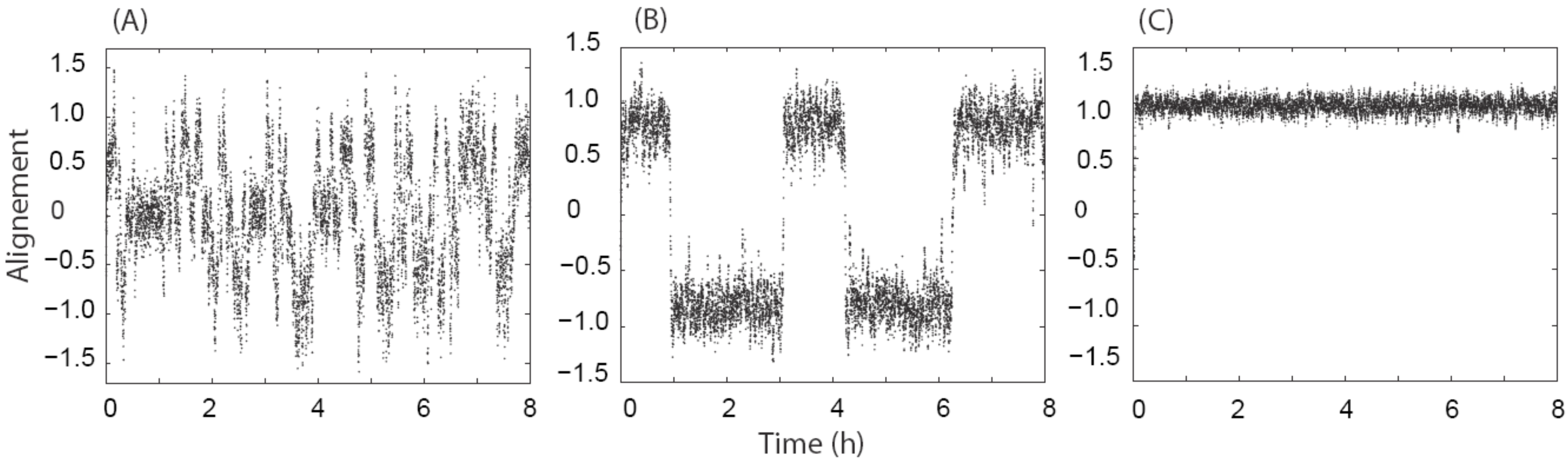


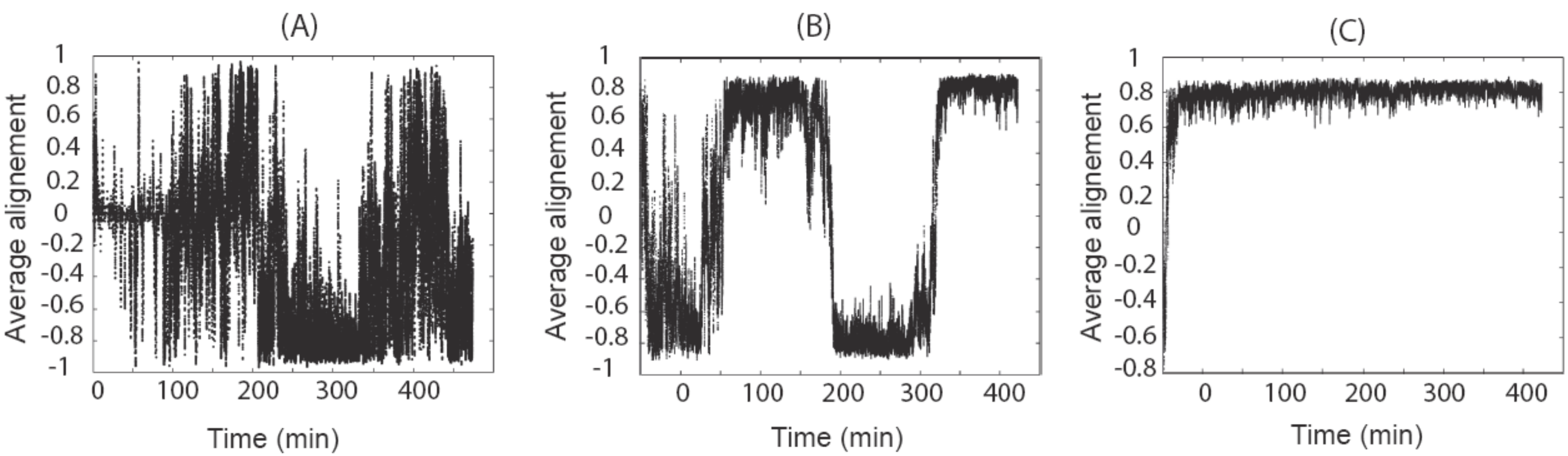
Buhl et al. (2006), *Science*



Buhl et al. (2006), *Science*

Model vs Experiment





Model vs Experiment