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DEPARTMENT OF MATHEMATICS
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Resit version
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Lab 2: Automata.

The deadline for this sheet is midnight Monday 31st of August.

Please submit hand-ins on Studentportalen. All code should be included. Please feel free to submit videos illustrating your results where appropriate, via Studentportalen or uploaded elsewhere. You may work in groups of size 1-4, and only one group member needs to submit the assignment. State clearly the members of the group.

2. Simulation of possible classifiers.

For the infinite line let each square independently be 1 with probability p and 0 otherwise. An automata is a classifier if with probability 1 after some number of iterations the infinite line is all 1s if $p > 1/2$ and all 0s if $p < 1/2$. It is unknown whether classifiers exist for the infinite line; this assignment investigates some proposed automata on finite lines with periodic boundary counditions - so that cells interact over the left and right boundaries.

Define the following two automata:

a **2-nbhd automata**. States are 0 and 1. A cell updates to 1 on next timestep if at least 3 of itself, two left neighbours and two right neighbours are 1's. Otherwise updates to 0. Equivalently - cell updates on next timestep to the majority state of itself, two left neighbours and two right neighbours.

b **GKL automata**. States are 0 and 1. If cell i is a 0 on next timestep updates to majority state of cells i, $i-1$ and $i-3$. If cell i is a 1 on next timestep updates to majority state of cells i, $i+1$, $i+3$.
Examples showing update of the underlined cell:
$101\underline{0}010 \to 1$, $110\underline{0}010 \to 0$ and $101\underline{1}010 \to 0$.

Note that the infinite line where all states are 0 is a fixed point of both automata. A fixed point is an arrangement of 0s and 1s on the infinite line, such that applying the automata, the next timestep returns the same arrangement of 0s and 1s. A period 2 point is one such that it is the same after two timesteps, but is not a fixed point.

1. For either the infinite line or for some finite N with periodic boundary conditions find examples of the following.

   (a) **2-nbhd automata**

      i. Two examples of fixed points (not all 1s or all 0s). The examples should not be translates of eachother.

      ii. A period 2 arrangement of 0s and 1s.

   (a) **GKL automata**

      i. A fixed point (not all 1s or all 0s)

      ii. A period 2 arrangement of 0s and 1s.

      iii. An arrangement of 0s and 1s with period > 2. (hint: $N = 30$ has period 22 and 26 points).

   (3 points).

2. For either the **2-nbhd** and **GKL** automata for a large-ish N (say 30-100). Simulate the 1D grid length N with periodic boundary conditions where each cell has a probability p of initially being in state 1. For an example simulation, plot how the cells typically look after 10 time steps, after 20 time steps and after 100 time steps. Simulate your model 100 times, generating the random initial state each time, and for 100 time steps and plot the average number of firing cells over time. Describe what typically happens in the system. Do this for $p = 0.4$ and $p = 0.6$. **(2 points)**.

3. Design your own automata as a candidate classifier, i.e. the aim is to have for an initial state with each cell 1 with probability p, after some number of timesteps that will be all 0s if $p < 1/2$ and all 1s if $p > 1/2$. Clearly explain the rule and repeat the previous question for this new rule. **(2 points)**

4. This question compares the three candidate automata (**2-nbhd**, **GKL** and your own) on the 1D grid of length N with periodic boundary conditions. For a range of p values (not $p = 1/2$), for each automata, plot the number of runs for which the automata takes the random initial state to the correct choice of all 0s or all 1s before some specified timestep t. Use your judgement to pick a timestep t, and also a number of runs, and which values of p so that you can compare the behaviour of the three candidate automata. **(3 points)**