Problem set up

- dimension n
- parameters e.g. k size of planted structure

p, q prob. of 'community' edges
$\quad$ " 'non-community' "

λ strength of signal

Each set of parameters, interested in behaviour at large n., or as $n \to \infty$. [WTS either EASY HARD IMPOSSIBLE]

<u>detection</u> H_0 : sample $G \sim Q_n$ 'null hypothesis'
H_1 : sample $G \sim P_n$ 'alt hypothesis'

test ϕ_n, $\phi_n(g) \in \{0,1\}$

'risk' ϕ

$$r(\phi) = \mathbb{P}_0(\phi(G)=1) + \mathbb{P}_1(\phi(G)=0)$$
$$= \sum_{g:\ \phi(g)=1} \mathbb{P}_0(G=g) + \sum_{g:\ \phi(g)=0} \mathbb{P}_1(G=g)$$

<u>Obs.</u> define ϕ_0, $\phi_0(g)=0 \ \forall g$, $r(\phi_0)=1$
$\Rightarrow$ risk '1' trivial

Defn

Say test ϕ_n achieves

<u>strong detection</u> if $r(\phi_n) \to 0$ as $n \to \infty$

<u>weak detection</u> if $\exists n_0, \exists \varepsilon > 0$ s.t.

$r(\phi_n) < 1 - \varepsilon \quad \forall n > n_0$.

Defn (EASY)

Say for $H_0 : Q_n(\alpha, \beta)$ vs $H_1 : P_n(\alpha, \beta)$

<u>strong detection</u> is EASY for parameters α, β.

if $\exists \phi_n$ polynomial time alg (or low deg alg)

s.t. $r(\phi_n) \to 0$ as $n \to \infty$.

(sim for weak detection).

Defn (POSSIBLE) "

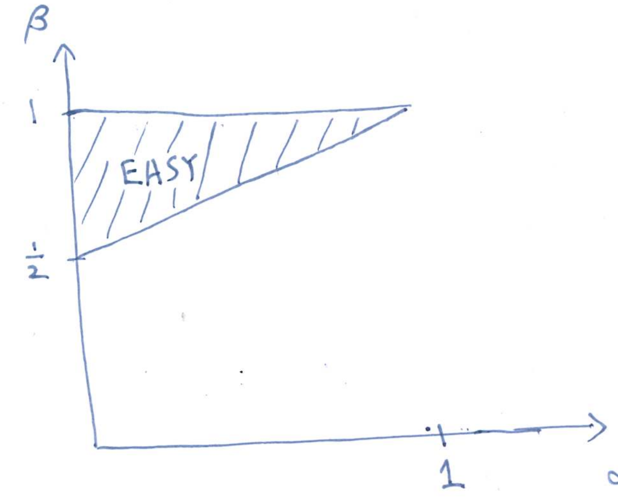
<u>strong detection</u> is POSSIBLE for " α, β

if $\exists \phi_n$ <u>any</u> alg s.t. $r(\phi_n) \to 0$ as $n \to \infty$.

[HARD if POSSIBLE and 'evidence of hardness'.]

Lemma

$k = n^{\beta}$



$\lambda = \Theta(n^{-\alpha})$

H_0 : $n \times n$ $X_{ij} \sim N(0,1)$ indep.

H_1: $n \times n$. $\sigma_i = \begin{cases} 1 & \text{w. prob } \frac{k}{n} \\ 0 & \text{o.w.} \end{cases}$

$$X_{ij} \mid \sigma \sim \begin{cases} N(\lambda, 1) & \text{if } \sigma_i = \sigma_j = 1 \\ N(0,1) & \text{o.w.} \end{cases}$$

strong detection for H_0 vs H_1 is EASY

for $\beta > \frac{\alpha}{2} + \frac{1}{2}$.

Recall • if Y_1, Y_2 indep r.v

then $var(Y_1 + Y_2) = var(Y_1) + var(Y_2)$

mean var

• if $Y_1 \sim N(a, b)$ $Y_2 \sim N(c, d)$

then $Y_1 + Y_2 \sim N(a+c, b+d)$.

$$\phi_{sum}(X) = \begin{cases} 1 & \text{if } \sum_{i,j} X_{ij} \geq \tau \quad \frac{k^2\lambda}{2} \\ 0 & \text{if } \sum_{i,j} X_{ij} < \tau \quad \frac{k^2\lambda}{2} \end{cases}$$

$$\sum_{i,j} X_{ij} \sim \begin{cases} N(0, n^2) & \text{under } H_0 \\ N(\lambda \tilde{k}^2, n^2) & \text{under } H_1 \end{cases}$$

where $\tilde{k} = \sum_i \sigma_i$

"diff in means >> sqrt var"

• intution want $\lambda k^2 >> n$ i.e. $n^{-\alpha} . n^{2\beta} >> 1$